AP Calculus AB Syllabus

Course Description

AP® Calculus AB is a yearlong, college-level course designed to prepare students for the Advanced Placement (AP) Calculus AB exam. Major topics of study in this full-year course include a review of pre-calculus; the use of limits, derivatives, definite integrals, and mathematical modeling of differential equations; and the applications of these concepts. Emphasis is placed on the use of technology to solve problems and draw conclusions. The course uses a multi-representative approach to calculus, with concepts and problems expressed numerically, graphically, verbally, and analytically. This course is aligned to the new College Board AP Calculus AB course description that was introduced in 2016.

Classroom Requirements

This AP course requires that a highly qualified math teacher be responsible for administering the course. This teacher is expected to interact with students regularly (either face to face or via chat and e-mail) and grade open-ended student responses. The teacher also should set up online discussions in the Collaboration Corner. In addition, he or she should provide instruction and coaching in calculator use as needed.

Course Materials

Textbook

Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic*, 5th ed. Boston: Pearson, 2016. **[CR 1]**

Graphing Calculator

On-screen teachers will use an online graphing calculator tool for demonstration purposes. **Students have access to a graphing calculator within the program and must have access to an AP-approved graphing calculator both in the classroom and at home.** Many assignments, projects, and practice exam questions will require the use of an AP-approved graphing calculator. Students will use graphing calculators to solve problems and to explore and interpret calculus concepts. **Students will be required to purchase a graphing calculator or check one out of the library. [CR 7]**
Classroom teachers are expected to demonstrate the use of graphing calculators to their students as needed.

Graphing Calculator Approach

Throughout the course, students are required to use multiple approaches to ensure their understanding of course content. In each lesson, students are required to use their graphing calculators to solve problems and to explore and interpret calculus concepts. **[CR 7]** In addition, students are required to graph problems by hand in order to supplement their understanding of calculus concepts.

Application problems requiring the use of analytical techniques in differential and integral calculus are included in each unit. Students will solve problems requiring numerical solutions both by hand and with their graphing calculators. Students are expected to check the reasonableness of their numerical solutions by using other approaches as well. They will also learn to use their graphing calculators to justify their conclusions and to support the solutions that they have developed using an analytical approach. **[CR 7]**

Every unit test mirrors the format of the AP exam. In each unit, students will answer multiple-choice questions and complete two free-response questions. The free-response questions will include questions that require students to provide reasoning for their work. Some of these questions will require a graphing calculator and some will not. Students will also complete two full-length practice exams at the end of the course. Students are required to use a graphing calculator on Part B of the multiple-choice questions and Part A of the free-response questions. **[CR 7]**

Examples of activities in which students use graphing calculators include:

Graph functions

Project: Designing a Roller Coaster: In this project, students use a graphing calculator to plot the parabola f(x). **[CR 7]**

Solve equations

Reading Lessons 1.1 and 8.2 (Areas in the Plane) Students graph linear functions using their calculators and utilize both the intersection function and the equation solver capabilities of the calculator to determine a solution to equations to three decimal places in Reading Lesson 1.1. Students also use the equation solver on the graphing calculator to determine points of intersection between two or more equations in order to graph the area enclosed by intersecting curves and to determine the endpoints needed for integration and then use the calculator's numerical integration function to find the area. **[CR 7]**

Project: Designing a Roller Coaster: In this project, students verify their answer by graphing f(x) on the graphing calculator. **[CR 7]**

Perform numerical differentiation

Differentiation Rules: In this lesson, students use the power rule to find derivatives, the product rule to find derivatives, and the quotient rule to find derivatives. Students calculate second derivatives and higher-order derivatives using rules of differentiation. Students also calculate instantaneous rates of change using the derivative. **[CR 7]**

Perform numerical integration

Trapezoidal Rule: In this lesson, students are to approximate the area under a curve using the trapezoidal rule. **[CR 7]**

Unit 6 Free Response Questions Students are given various functions that they cannot integrate analytically using the techniques they have learned. They must instead use the graphing calculator's numerical integration feature to compute definite integrals of these functions on stated intervals. **[CR 7]**

Explore or interpret calculus concepts

Students use graphing calculators to investigate concepts connecting calculus to their previous experiences in mathematics. For example, students use their graphing calculators to explore how changing the number of subdivisions will affect the accuracy of Riemann sums. Students graph a given function on their graphing calculator and determine the approximate area under the curve for a given closed interval, [a, b], using a left Riemann sum with 4, 8, and 12 subdivisions. Students are asked to

explain which sum results in a more accurate approximation and justify their answer, using appropriate mathematical language. Students will repeat the process for a right and midpoint Riemann sum. **[CR 7]**

Across all lessons in the course, students are required to use their graphing calculator to solve problems. Students will use their graphing calculators experimentally to determine possible solutions to problems. They will also learn to use their calculators to justify their conclusions and to support the solutions that they have developed using an analytical approach.

Newton's Method, Linearization, and Differentials: In this lesson, students are asked to apply Newton's method to approximate zeros of a function and to use linearization to approximate tangent lines. The related work in Reading Lesson 5.5 and Practice Problems 5.5 walks students through the steps to perform the calculations of Newton's Method using a graphing calculator. **[CR 7]**

Reading Lesson 2.1 and Practice Problems 2.1: In this work, students explore how graphing calculator generated graphs and tables for functions can be used to determine limits. Students graph functions to find limits at given points and then verify their answers algebraically. **[CR 7]**

Course Outline – Big Ideas and Required Course Content

The Course Outline – Big Ideas and Required Course Content shows the overall structure of the course by unit, highlights how the Big Ideas are incorporated, and identifies the required content outlined in each of the units described in the Course and Exam Description (CED) **[CR 2]**

Unit 1: Pre-Calculus Review

Big Ideas and CED Alignment:

Pre-Calculus Review covers key concepts from Pre-Calculus which are necessary to be successful in AP Calculus AB. This unit is not aligned to the CED units or Big Ideas.

Unit 2: Limits and Continuity

Big Ideas and CED Alignment:

Limits and Continuity covers required course content from Unit 1 in the CED, including the Big Ideas of Limits, Change, and Analysis of Functions. This work includes, but is not limited to

- Using limits to understand change,
- Interpreting instantaneous and average rates of change,
- Representing limits using correct notation,
- Expressing limits graphically, numerically, and analytically,
- Estimating limits of functions,
- Determining one-sided limits,
- Determining the limits of functions using properties and limit theorems,
- Using equivalent expressions or the squeeze theorem to determine limits,
- Identifying types of discontinuities,
- Determining when equivalent expressions can be created that allow for the removal of discontinuities,
- Justifying conclusions about continuity at a point using the definition,
- Determining intervals over which a function is continuous,
- Solving for parameters that make discontinuous functions continuous, when possible,
- Interpreting the behavior of functions using limits involving infinity, and
- Explaining the behavior of a function on an interval using the Intermediate Value Theorem.

Unit 3: Derivatives

Big Ideas and CED Alignment:

Derivatives covers required course content from Unit 2 in the CED, including the Big Idea of Limits, Change, and Analysis of Functions. This work includes, but is not limited to

- Determining average rates of change using difference quotients,
- Comparing and contrasting the slope of the secant line to the slope of the tangent line,
- Representing the derivative of a function as the limit of a difference quotient,
- Defining the derivative using the formal definition,
- Understanding that the derivative can be represented graphically, numerically, analytically, and verbally,
- Determining the equation of a line tangent to a curve at a given point,
- Estimating derivatives from information given in tables or graphs or using technology,
- Explaining the relationship between differentiability and continuity,
- Using derivative rules to differentiate sums, differences, and constant multiples of functions,
- Combining the power rule with sum, difference, and constant multiple properties to find the derivatives for polynomial functions,
- Finding the derivatives for sine, cosine, exponential, and logarithmic functions,
- Interpreting a limit as a definition of a derivative,
- Recognizing that in some cases an expression for the definition of the derivative of a function whose derivative is known offers a strategy for determining a limit,
- Calculating derivatives of products and quotients of differentiable functions using the product rule or the quotient rule,
- Differentiating tangent, cotangent, secant, and cosecant functions by rearranging them using identities and using derivative rules, and
- Using the derivative to solve rectilinear motion problems involving position, speed, velocity, and acceleration.

Unit 4: More Derivatives

Big Ideas and CED Alignment:

More Derivatives covers required course content from Unit 3 in the CED, including the Big Ideas of Analysis of Functions. This work includes, but is not limited to

- Calculating derivatives of compositions of differentiable functions using the chain rule,
- Calculating derivatives of implicitly defined functions,
- Calculating derivatives of inverse and inverse trigonometric functions,
- Selecting an appropriate procedure for calculating derivatives, and
- Determining higher order derivatives of a function using a variety of notations.

Unit 5: Applications of Derivatives

Big Ideas and CED Alignment:

Applications of Derivatives covers required course content from Units 4* and 5 in the CED, including the Big Ideas of Limits, Change, and Analysis of Functions. This work includes, but is not limited to

- Interpreting the meaning of a derivative in context,
- Using the derivative to solve problems involving rates of change in applied contexts,
- Calculating related rates in applied contexts,
- Using the chain rule for differentiating variables in a related rates problem with respect to the same independent variable,
- Interpreting related rates in applied contexts,
- Approximating a value on a curve using the equation of a tangent line,
- Justifying conclusions about functions by applying the Mean Value Theorem over an interval,
- Justifying conclusions about functions by applying the Extreme Value Theorem,
- Justifying conclusions about the behavior of a function based on the behavior of its derivatives utilizing the first and second derivative tests,
    - determining where the function is increasing or decreasing
    - determining the location of relative (local) extrema of the function
    - drawing conclusions about absolute (global) extrema of a function on a closed interval
    - determining the concavity of a function on an open interval
    - locating points of inflection for the graph of a function
    - determining whether a critical point is the location of a relative maximum or minimum
- Using derivatives to identify key features of functions and to represent functions graphically, numerically, and analytically,
- Calculating and interpreting minimum and maximum values in applied contexts or analysis of functions (optimization problems),
- Determining critical points of implicit relations, and
- Justifying conclusions about the behavior of an implicitly defined function using derivatives.

*Straight-line motion is covered in Unit 3, and L'Hopital is covered in Unit 8.

Unit 6: Definite Integrals

Big Ideas and CED Alignment:

Definite Integrals covers required course content from Unit 6* in the CED, including the Big Ideas of Limits, Change, and Analysis of Functions. This work includes, but is not limited to

- Interpreting the meaning of areas associated with the graph of a rate of change in context,
- Approximating a definite integral using geometric and numerical methods,
    - using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum
    - using either uniform or nonuniform partitions
    - using numerical methods, with or without technology
    - using area formulas for triangle, rectangles, circles, or trapezoids
- Determining whether an approximation for a definite integral is an underestimate or overestimate for the value of the definite integral,
- Interpreting the limiting case of the Riemann sum as a definite integral,
- Representing the limiting case of the Riemann sum as a definite integral,
- Translating a definite integral into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral,
- Representing accumulation functions using definite integrals,
- Calculating a definite integral using areas and properties of definite integrals,

    - including the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals
- Interpreting areas under the x-axis as negative values and apply techniques required to find total area
- Extending the definition of the definite integral to functions with removable or jump discontinuities,
- Evaluating definite integrals analytically using the Fundamental Theorem of Calculus,
- Determining antiderivatives of functions and indefinite integrals, and
- Developing the skill of selecting an appropriate procedure for antidifferentiation.

*Concepts of integration using substitution are found in Unit 7.

Unit 7: Mathematical Modeling Using Differential Equations

Big Ideas and CED Alignment:

Mathematical Modeling Using Differential Equations covers required course content from Unit 7 in the CED, including the Big Idea of Analysis of Functions. This work includes, but is not limited to

- Using substitution of variables as a technique for finding antiderivatives,
- Interpreting verbal statements of problems as differential equations involving a derivative expression,
- Verifying solutions to differential equations,
- Estimating solutions to differential equations,
- Recognizing a slope field as a graphical representation of a differential equation on a finite set of points in the plane,
- Determining general solutions to differential equations graphically and analytically,
- Determining particular solutions to differential equations,
- Noting domain restrictions to solutions to differential equations when appropriate,
- Interpreting the meaning of a differential equation and its variables in context, including motion along a line and exponential growth and decay, and
- Determining general and particular solutions for problems involving differential equations in context.

Unit 8: Applications of Definite Integrals

Big Ideas and CED Alignment:

Applications of Definite Integrals covers required course content from Unit 8 in the CED, including the Big Idea of Change. This work includes, but is not limited to

- Determining limits of functions that result in indeterminate forms using L'Hospital's Rule
- Determining the average value of a function using definite integrals,
- Using definite integrals to determine values for positions and rates of change in problems involving rectilinear motion,
- Interpreting the meaning of a definite integral in accumulation problems,
- Determining net change using definite integrals in applied contexts,
- Calculating areas of regions in the plane using the definite integral,
    - using functions of either x or y
    - using a sum of two or more definite integrals or by evaluating a definite integral of the absolute value of the difference of two functions

- Calculating volumes of solids with known cross sections using definite integrals, and
    - determining volumes of solids with square and rectangular cross sections
    - determining volumes of solids with triangular cross sections
    - determining volumes of solids with semicircular and other geometrically defined cross sections
- Calculating volumes of solids of revolution using definite integrals.
    - using the disc method to calculate volumes of solids of revolution around the x- or y-axis or around any horizontal or vertical line in the plane
    - using the washer method to calculate volumes of solids of revolution around the x- or y-axis or around any horizontal or vertical line whose cross sections are ring shaped

Course Outline – Lessons and Activities

The Course Outline shows the overall structure of the course with units and lessons and highlights sample activities.

Unit 1: Pre-Calculus Review

Lessons:

- Intro to AP Calculus
- Lines
- Functions and Graphs **[CR 6 MP 4.D]**
- Exponential Functions
- Parametric Equations
- Functions and Logarithms
- Trigonometric Functions

Sample Activities:

Students will review using a graphing calculator to solve systems of linear equations in Reading Lesson 1.1 and its related problem set in the text. **[CR 7]**

Unit 2: Limits and Continuity

Lessons:

- Introduction to Limits and Continuity **[CR 3 MP 1.A]**
- Rates of Change, Limits, and the Squeeze Theorem **[CR 3 MP 1.A]**
- Limits Involving Infinity and Vertical and Horizontal Asymptotes
- Continuous Functions and the Intermediate Value Theorem **[CR 3 MP 1.A] [CR 4 MP 2.A] [CR 5 MP 3.C] [CR 5 MP 3.D]**
- Slope, Tangent Line, and Normal Line

Sample Activities:

Students will practice computing limits both algebraically and graphically. They will also practice analyzing functions for intervals of continuity and points of discontinuity. They will use limits to find instantaneous rates of change, slopes of tangent lines, and sensitivity to change. In addition, students will solve problems using the extreme value theorem. **[CR 3 MP1.A] [CR 4 MP 2.A] [CR 4 MP 2.E] [CR 5 MP 3.B]**

Students communicate their understanding of the Intermediate Value Theorem using appropriate mathematical language by solving problems where they have to determine if the criteria for the theorem has been met, and then use the theorem to solve when it has. **[CR 6 MP 4.A]**

Students will identify and apply the appropriate limit property and procedure based on the structure of the functions or limit information given in Reading Lesson 2.1 and its related problem set in the text. **[CR 3 MP 1.C]**

Students verify that the hypothesis is satisfied for the Intermediate Value Theorem and then apply the theorem to solve for unknown values. **[CR 5 MP 3.C] [CR 5 MP 3.D]**

Unit 3: Derivatives

Lessons:

- Introduction to Derivatives
- Derivatives of Functions **[CR 3 MP 1.B] [CR 6 MP 4.C]**
- Derivatives and Continuity **[CR 3 MP 1.A]**
- Differentiation Rules **[CR 3 MP 1.E]**
- Applications of Derivatives
- Differentiating Trigonometric Functions

Sample Activities:

Given the equation for the movement of a particle, students will find the particle's average velocity over an interval, derive an expression for the particle's velocity, and determine when the particle changes direction. Students must show their work and provide a written explanation of the thought process used to solve the problem. **[CR 3 MP 1.B] [CR 4 MP 2.C] [CR 4 MP 2.E] [CR 5 MP 3.B]**

Students will practice using a graphing calculator to perform numerical differentiation in Reading Lesson 3.2 and its related problem set in the text. **[CR 7]**

Students will practice graphing f from f', and vice versa. **[CR 6 MP 4.D]**

Given a real-world problem involving two people traveling along a path, students will use a data table to estimate the value of a derivative, calculate the people's acceleration and velocity, and determine at what time the distance between the two reaches a certain point. Students will relate the big idea of velocity and time given in a table (numerical) also given in function form. **[CR 3 MP 1.B] [CR 3 MP 1.D] [CR 4 MP 2.C] [CR 4 MP 2.C] [CR 5 MP3.F]**

Designing a Roller Coaster Project: Students will design the first drop of a new roller coaster. First, they will conduct research and determine the optimal slopes for the ascent and descent portions. Then, they will write and solve equations that connect the two segments with a parabola and ensure that the track remains smooth. Next, they will sketch the lines representing the ascent and descent and the parabola that connects the two. That is, students will connect the algebraic equations representing the slopes of the curve to the graphical representation of the curve. Finally, they will use a calculator to plot the parabola using the equations they developed and compare that plot to their sketch. Students will be required to present their findings both graphically and numerically, translating between the two representations to justify their conclusions. They must provide an annotated diagram of their design as well as step-by-step written solutions and justifications to all of the problems to be solved **[CR 4 MP 2.C] [CR 4 MP 2.D] [CR 5 MP 3.B] [CR 5 MP 3.C] [CR 5 MP3.E] [CR 5 MP3.F] [CR 8]**

As a part of the project, students communicate their understanding of differentiable functions by constructing a polynomial to model a real world scenario and are requited to explain how they know it is differentiable over the necessary interval through a written response task. Students are asked to verify their results using the graphing calculator and connect the concepts given analytically in function form and in graphic form, again through a written response task. They will be checked for their use of appropriate mathematical language in this work. **[CR 6 MP 4.A]**

Unit 4: More Derivatives

Lessons:

- Introduction to More Derivatives **[CR 3 MP 1.B]**
- Differentiating Functions Using the Chain Rule **[CR MP 1.E]**
- Differentiating Functions Using Implicit Differentiation
- Differentiating Functions Containing Inverse Functions and Inverse Trigonometric Functions
- Differentiating Exponential and Logarithmic Functions

Sample Activities:

Given a real-world problem involving water flowing in and out of a reservoir, students will calculate average rate of change, use the chain rule to calculate a derivative, and compare rates of change to the average rate of change. Students must show their work and provide a written explanation of the thought process used to solve the problem. **[CR 3 MP 1.B] [CR 3 MP 1.D] [CR 4 MP 2.B] [CR 5 MP 3.C] [CR 5 MP3.F]**

Given an equation, students will use the chain rule to find its derivative and the equation of the line tangent to the curve at a given point. Students must show their work and provide a written explanation of the thought process used to solve the problem. **[CR 3 MP 1.B] [CR 4 MP 2.E]**

Unit 5: Applications of Derivatives

Lessons:

- Introduction to Applications of Derivatives
- Relative and Absolute Extrema **[CR 4 MP 2.D]**
- The Mean Value Theorem **[CR 3 MP 1.B] [CR 5 MP 3.C] [CR 5 MP 3.D]**
- The First and Second Derivative Tests **[CR 4 MP 2.C]**
- Application Problem-Solving
- Newton's Method, Linearization, and Differentials
- Applications of Implicit Differentiation

Sample Activities:

Students verify that the hypothesis is satisfied for the Mean Value Theorem and then apply the theorem to solve for unknown values. **[CR 5 MP 3.C] [CR 5 MP 3.D]**

Using the graph of the derivative of a function, students will determine key features of the function, such as increasing and decreasing intervals, local extrema, points of inflection, and concavity intervals. Students solve application problems, including related rates and optimization problems. Within this work students move between analytical, graphical, and verbal representations of functions. **[CR 3 MP 1.B] [CR 4 MP 2.b] [CR 4 MP 2.C] [CR 4 MP 2.E] [CR 6 MP 4.D]**

Graphically students understand that extrema are turning points. Algebraically it is where the derivative is equal to zero. **[CR 4 MP 2.D]**

Students will use implicit differentiation to solve a real-world related rate problem. Students must show their work and provide a written explanation of the thought process used to solve the problem. **[CR 3 MP 1.B] [CR 4 MP 2.B] [CR 4 MP 2.C] [CR 5 MP3.F]**

Students are asked to determine if a given piecewise, rational, or polynomial function satisfies the hypotheses of the Mean Value Theorem using precise mathematical language of continuity and differentiability. If the criteria has been met, they are then asked to apply the theorem and interpret their results. **[CR 6 MP 4.A]**

Soda Can Optimization Project: Students will solve an optimization problem to find the most economical dimensions for a soda can, given a specific volume that the can must hold. Students will provide step-by-step written solutions, diagrams, and justifications to all of the problems to be solved to achieve the can's optimization. In addition, students will conduct research to identify real-world examples and non-examples of their optimized can design and write a report about their findings. Students will present their findings to the class. **[CR 3 MP 1.B] [CR 3 MP 1.E] [CR 4 MP 2.B] [CR 4 MP 2.C] [CR 5 MP3.F] [CR 8]**

Unit 6: Definite Integrals

Lessons:

- Introduction to Definite Integrals
- Estimating with Finite Sums **[CR 3 MP 1.F]**
- Definite Integrals **[CR 3 MP 1.C]**
- Definite Integrals and Antiderivatives
- The Fundamental Theorem of Calculus (Parts I and II) **[CR 3 MP 1.C]**
- The Trapezoidal Rule **[CR 3 MP 1.E]**

Sample Activities:

Students will practice estimating with finite sums and finding definite integrals and antiderivatives, including in problems with initial conditions. They will also calculate the areas under curves using various techniques, including rectangular approximation methods and the trapezoidal rule. In addition, students will calculate the average value of a function over a closed interval and use this concept to understand the mean value theorem. Students will interpret the area under a graph as a net accumulation of a rate of change. **[CR 3 MP 1.C] [CR 3 MP 1.F] [CR 4 MP 2.A] [CR 4 MP 2.E] [CR 5 MP 3.Bb] [CR 6 MP 4.E] [CR 6]**

Students will apply the first part of the fundamental theorem of calculus to solve problems and the second part to calculate integrals analytically. **[CR 3 MP 1.C]**

Students will practice using a graphing calculator to perform numerical integration in Reading Lesson 6.2 and its related problem set in the text. **[CR 7]**

Students will communicate the concept of a definite integral using both summation and integral notation. In doing so, they will explain how to translate one notation into the other using appropriate

mathematical language, recognizing that they both represent the same mathematical concept. **[CR 6 MP 4.A]**

Analyzing Driving Data: In this project, students will use calculus to test the notion that the area under a velocity/time curve equates to the total distance traveled. Students will apply the procedures of Riemann sum approximations, using the mid-point and trapezoidal approximations to estimate the area under the curve. They will compare their calculated estimates to the driving data and use error analysis to account for discrepancies. **[CR 3 MP 1.D] [CR 5 MP 3.F] [CR 8]**

Within this project, students communicate their understanding of the Fundamental Theorems of Calculus by using real world data to create accumulation tables and/or graphs. They must demonstrate an understanding of the notation for multiple integration techniques and produce an error analysis by comparing their experimental results and theoretical results from the theorems. Students are required to collect data, perform appropriate calculations based on the concepts of velocity and distance, apply estimation of area under the curve techniques connected to geometry and integration, and justify their calculations and errors through written response. They will be checked for their use of appropriate mathematical language. **[CR 6 MP 4.A]**

Unit 7: Mathematical Modeling Using Differential Equations

Lessons:

- Introduction to Mathematical Modeling Using Differential Equations
- Slope Fields
- Antidifferentiation by Substitution **[CR 3 MP 1.E]**
- Exponential Growth and Decay

Sample Activities:

Students will practice using slope fields to analyze and construct solutions to differential equations. They will also practice finding antiderivatives of functions and solving separable differential equations, including those coming from problems involving exponential growth, and exponential decay. **[CR 3 1.C] [CR 4 MP 2.A] [CR 4 MP 2.C] [CR 4 MP 2.E] [CR 3 MP 1.B]**

Students will match slope fields (graphical) to differential equations (analytical). They then use the slope field to sketch the graph of the particular solution through a highlighted point. Conversely, students are asked to match a differential equation (analytical) to a given slope field (graphical). **[CR 4 MP 2.A] [CR 4 MP 2.C] [CR 4 MP 2.E]**

Students will express differential equations in algebraic and graphic forms utilizing slope fields to represent the family of solutions in Reading Lesson 7.1 and its related problem set in the text. **[CR 4 MP 2.C]**

Unit 8: Applications of Definite Integrals a

Lessons:

- Introduction to Applications of Definite Integrals
- Integral as Net Change
- Areas in the Plane
- Volumes **[CR 6 MP 4.B]**
- Applications from Science and Statistics
- L'Hopital's Rule and Other Applications **[CR 4 MP 2.A]**

Sample Activities:

Students will practice applying the definite integral to solve problems involving accumulation, displacement, total distance, net change, areas, work, fluid pressure, and probability distribution, and justify their findings in writing. **[CR 3 MP 1.C] [CR 4 MP 2.C] [CR 6 MP 4.B]**

Students will calculate the volume of a solid generated by revolving a line or curve around a given line and by revolving a region bounded by two or more lines or curves. They will also calculate the volume of a solid using cross-sections. **[CR 3 1.C] [CR 6 MP 4.B]**

Students will make connections between Riemann sums and definite integrals by including the Riemann sum notation as part of the answer along with the definite integral for some problems. In this work, students will work with Riemann sums with the number of subdivisions tending to infinity. **[CR 4 MP 2.A] [CR 6 MP 4.C]**

Students are asked to translate the limit of a Riemann sum with the number of subdivisions tending to infinity into the corresponding definite integral when given endpoints of integration and for sums where they must determine the endpoints of integration themselves. **[CR 6 MP 4.C]**

Students are given both a finite and infinite Riemann sum with proper summation notation and must determine the corresponding definite integral using proper integral notation. Conversely, students are given definite integrals and must use the appropriate mathematical symbols to determine the accumulator function. **[CR 6 MP 4.C]**

Students will practice using L'Hopital's rule to evaluate limits of indeterminate forms and compare the growth rates of functions. **[CR 3 1.C] [CR 4 MP 2.A] [CR 4 MP 2.E] [CR 5 MP 3.B] [CR 6]**

Calculating the Volume of an Object: In this project, students will coordinate piece-wise functions with integration techniques to calculate the volume and surface area of real-world objects. Students will model their object using functions and then calculate volume with the disk method and surface area using the appropriate formula. Students will also perform an error analysis to account for any discrepancies between their calculations and the actual volume of the object. **[CR 5 MP3.F] [CR 8]**

Unit 9: Course Review and AP Exam Preparation

The course concludes with a unit focused on preparing students for the AP exam.