Geometric Constructions

In this lesson, you learned how a compass and straightedge are used to create constructions related to circles. In this assignment, you will use those tools to complete those constructions on your own.

As you complete the assignment, keep this question in mind:

How do you perform constructions related to circles?

Directions

Complete each of the following constructions, reading the directions carefully as you go. Be sure to show all work and insert an image of each construction. If you are unable to take and insert screenshots of the constructions, print this activity sheet and create them by hand using a compass and straightedge.

Your teacher will give you further directions about how to submit your work. You may be asked to upload the document, e-mail it to your teacher, or print it and hand in a hard copy.

Now, let's get started!

Step 1: Construct a circle through three points not on a line.

a. Construct a circle through three points not on a line using the construction tool. Insert a screenshot of the construction here. Alternatively, construct a circle through three points not on a line by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)

Step 2: Construct regular polygons inscribed in a circle.

a. Construct an equilateral triangle inscribed in a circle using the construction tool. Insert a screenshot of the construction here. Alternatively, construct an equilateral triangle inscribed in a circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)

b. Construct a regular hexagon inscribed in a circle using the construction tool. Insert a screenshot of the construction here. Alternatively, construct a regular hexagon inscribed in a circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)

c. Construct a square inscribed in a circle using the construction tool. Insert a screenshot of the construction here. Alternatively, construct a square inscribed in a circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)

Step 3: Construct tangent lines to a circle.

a. Construct a tangent to a circle through a point on the circle using the construction tool. Insert a screenshot of the construction here. Alternatively, a tangent to a circle through a point on the circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)

b. Construct a tangent to a circle through a point outside the circle using the construction tool. Insert a screenshot of the construction here. Alternatively, construct a tangent to a circle through a point outside the circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)

Student Guide

Geometric constructions date back thousands of years to when Euclid, a Greek mathematician known as the "Father of Geometry," wrote the book *Elements*. In *Elements*, Euclid formulated the five postulates that form the base for Euclidean geometry. To create all the figures and diagrams, Euclid used construction techniques extensively. A compass and straightedge are used to create constructions. A compass is used to draw circles or arcs and a straightedge is used to draw straight lines.

As you complete the task, keep these questions in mind:

How do you perform constructions related to circles? What theorems and explanations can be used to justify these constructions?

In this task, you will apply what you have learned in this lesson to answer these questions.

Directions

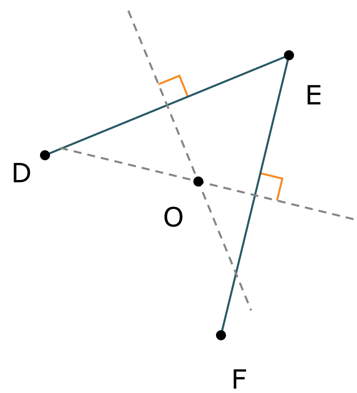
Complete each of the following tasks, reading the directions carefully as you go. Be sure to show all work where indicated, including inserting images of constructions created using the tool. If you are unable to take and insert screenshots of the construction tool, print this activity sheet and create the constructions by hand using a compass and straightedge.

In addition to the answers you determine, you will be graded based on the work you show, or your solution process. So, be sure to show all your work and answer each question as you complete the task. Type all your work into this document so you can submit it to your teacher for a grade. You will be given partial credit based on the work you show and the completeness and accuracy of your explanations.

Now, let's get started!

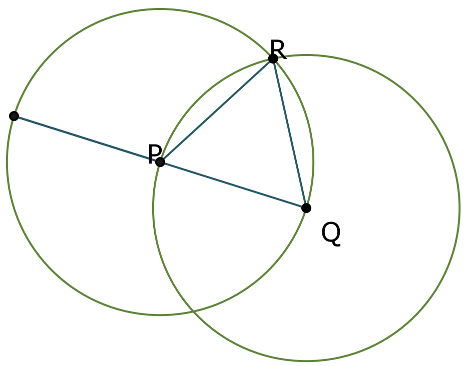
Step 4: Construct a circle through three points not on a line.

a. Points D, E, and F are not in a line. To construct a circle through points D, E, and F, begin by drawing line segments $\overline{DE}$ and $\overline{EF}$. Then construct the perpendicular bisectors of $\overline{DE}$ and $\overline{EF}$, and name the point of intersection of the perpendicular bisectors O. How do you know that point O is the center of the circle that passes through the three points? (10 points)

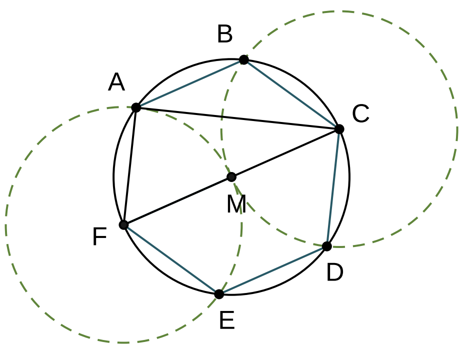


Step 5: Construct regular polygons inscribed in a circle.

a. While constructing an equilateral triangle or a regular hexagon inscribed in a circle, you may have noticed that several smaller equilateral triangles are formed, like △PQR shown in the figure below. Explain why △PQR is an equilateral triangle. (5 points)



b. The completed construction of a regular hexagon is shown below. Explain why △ACF is a 30˚-60˚-90˚ triangle. (10 points)



c. If you are given a circle with center C, how do you locate the vertices of a square inscribed in circle C? (5 points)

Step 6: Construct tangent lines to a circle.

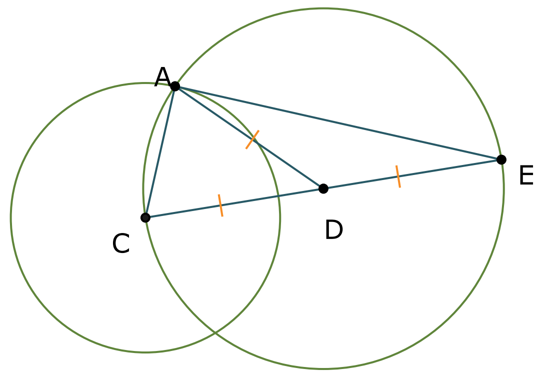
a. $\overline{JL}$ is a diameter of circle K. If tangents to circle K are constructed through points L and J, what relationship would exist between the two tangents? Explain. (5 points)

b. The construction of a tangent to a circle given a point outside the circle can be justified using the second corollary to the inscribed angle theorem. An alternative proof of this construction is shown below. Complete the proof. (5 points)

Given: Circle C is constructed so that CD = DE = AD; $\overline{CA}$ is a radius of circle C.

Prove: $\overline{AE}$ is tangent to circle C.

Student Guide



Statements	Reasons
1. Circle C is constructed so that CD = DE = AD; $\overline{CA}$ is a radius of circle C.	1. Given
2. $\overline{CD} \cong \overline{DE} \cong \overline{AD}$	2. Definition of congruence
3. $\triangle ACD$ is an isosceles triangle; $\triangle ADE$ is an isosceles triangle.	3.
4. $m\angle CAD + m\angle DCA + m\angle ADC = 180°$; $m\angle DAE + m\angle AED + m\angle EDA = 180°$	4.
5.	5. Isosceles triangle theorem
6. $m\angle CAD = m\angle DCA$; $m\angle DAE = m\angle AED$	6. Definition of congruence
7. $m\angle CAD + m\angle CAD + m\angle ADC = 180°$; $m\angle DAE + m\angle DAE + m\angle EDA = 180°$	7. Substitution property
8. $2(m\angle CAD) + m\angle ADC = 180°$; $2(m\angle DAE) + m\angle EDA = 180°$	8. Addition
9. $m\angle ADC = 180° - 2(m\angle CAD)$; $m\angle EDA = 180° - 2(m\angle DAE)$	9.
10. $\angle ADC$ and $\angle EDA$ are a linear pair.	10.

Student Guide

11.	11. Linear pair postulate
12. $m\angle ADC + m\angle EDA = 180°$	12. Definition of supplementary angles
13. $180° - 2(m\angle CAD) + 180° - 2(m\angle DAE) = 180°$	13. Substitution property
14. $360° - 2(m\angle CAD) - 2(m\angle DAE) = 180°$	14. Addition
15. $-2(m\angle CAD) - 2(m\angle DAE) = -180°$	15. Subtraction property
16. $m\angle CAD + m\angle DAE = 90°$	16.
17. $m\angle CAD + m\angle DAE = m\angle CAE$	17. Angle addition postulate
18.	18. Substitution property
19. $\angle CAE$ is a right angle.	19. Definition of right angle
20.	20. Definition of perpendicular
21. $\overline{AE}$ is tangent to circle C.	21.