Geometric Constructions

In this lesson, you learned how a compass and straightedge are used to create constructions related to circles. In this assignment, you will use those tools to complete those constructions on your own.

As you complete the assignment, keep this question in mind:

How do you perform constructions related to circles?

Directions

Complete each of the following constructions, reading the directions carefully as you go. Be sure to show all work and insert an image of each construction. If you are unable to take and insert screenshots of the constructions, print this activity sheet and create them by hand using a compass and straightedge.

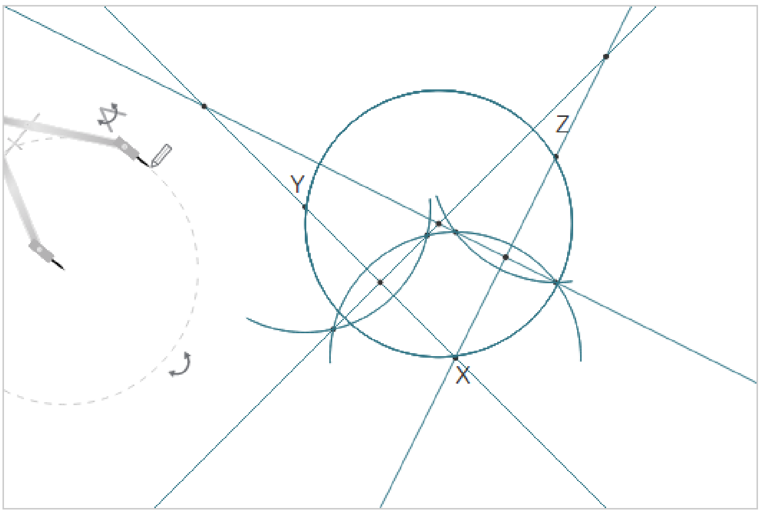
Your teacher will give you further directions about how to submit your work. You may be asked to upload the document, e-mail it to your teacher, or print it and hand in a hard copy.

Now, let's get started!

Answer Key

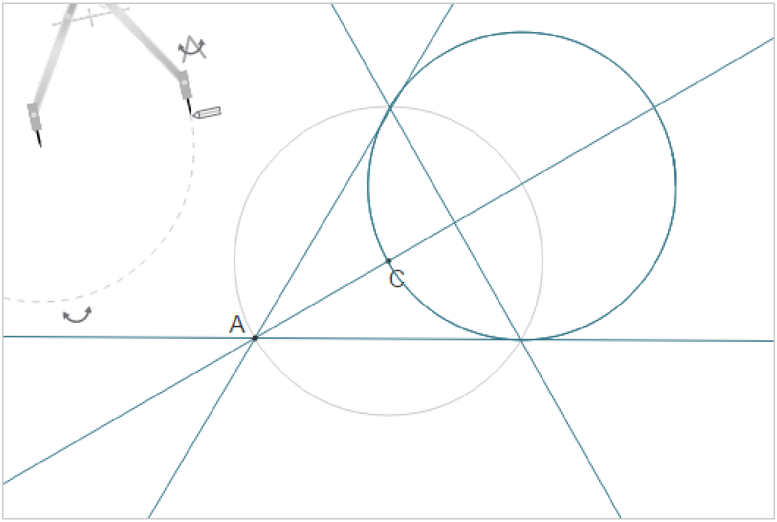
Step 1: Construct a circle through three points not on a line.

a. Construct a circle through three points not on a line using the construction tool. Insert a screenshot of the construction here. Alternatively, construct a circle through three points not on a line by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)

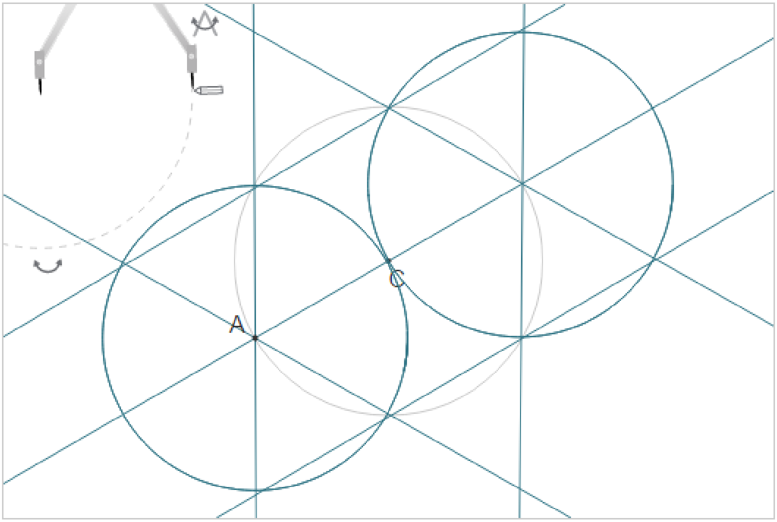


Step 2: Construct regular polygons inscribed in a circle

a. Construct an equilateral triangle inscribed in a circle using the construction tool. Insert a screenshot of the construction here. Alternatively, construct an equilateral triangle inscribed in a circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)



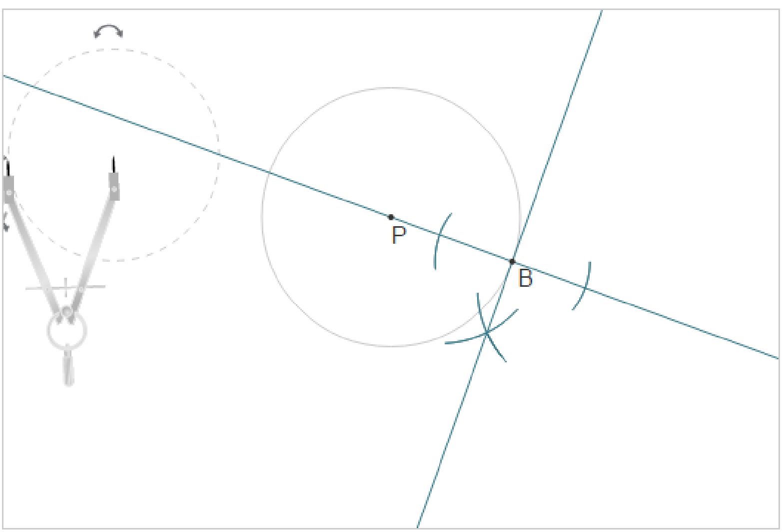
b. Construct a regular hexagon inscribed in a circle using the construction tool. Insert a screenshot of the construction here. Alternatively, construct a regular hexagon inscribed in a circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)



c. Construct a square inscribed in a circle using the construction tool. Insert a screenshot of the construction here. Alternatively, construct a square inscribed in a circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)

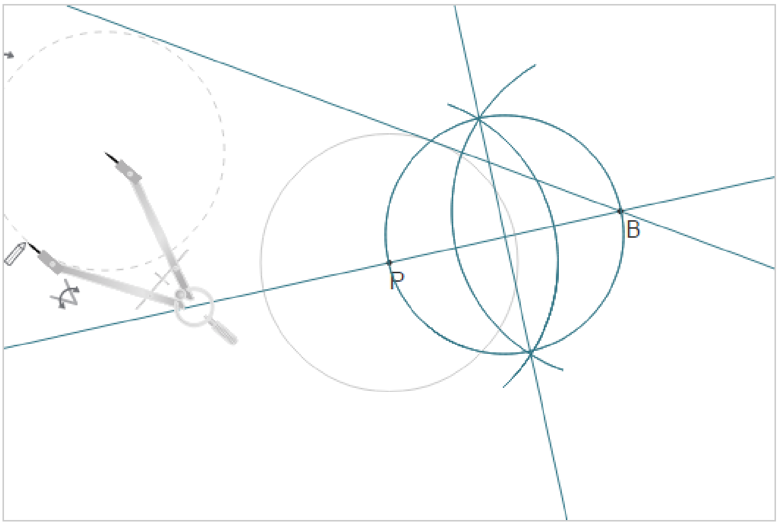
Step 3: Construct tangent lines to a circle.

a. Construct a tangent to a circle through a point on the circle using the construction tool. Insert a screenshot of the construction here. Alternatively, a tangent to a circle through a point on the circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)



Answer Key

b. Construct a tangent to a circle through a point outside the circle using the construction tool. Insert a screenshot of the construction here. Alternatively, construct a tangent to a circle through a point outside the circle by hand using a compass and straightedge. Leave all circle and arc markings. (10 points)



Answer Key

Geometric Constructions

Geometric constructions date back thousands of years to when Euclid, a Greek mathematician known as the "Father of Geometry," wrote the book *Elements*. In *Elements*, Euclid formulated the five postulates that form the base for Euclidean geometry. To create all the figures and diagrams, Euclid used construction techniques extensively. A compass and straightedge are used to create constructions. A compass is used to draw circles or arcs and a straightedge is used to draw straight lines.

As you complete the task, keep these questions in mind:

How do you perform constructions related to circles? What theorems and explanations can be used to justify these constructions?

In this task, you will apply what you have learned in this lesson to answer these questions.

Directions

Complete each of the following tasks, reading the directions carefully as you go. Be sure to show all work where indicated, including inserting images of constructions created using the tool. If you are unable to take and insert screenshots of the construction tool, print this activity sheet and create the constructions by hand using a compass and straightedge.

In addition to the answers you determine, you will be graded based on the work you show, or your solution process. So, be sure to show all your work and answer each question as you complete the task. Type all your work into this document so you can submit it to your teacher for a grade. You will be given partial credit based on the work you show and the completeness and accuracy of your explanations.

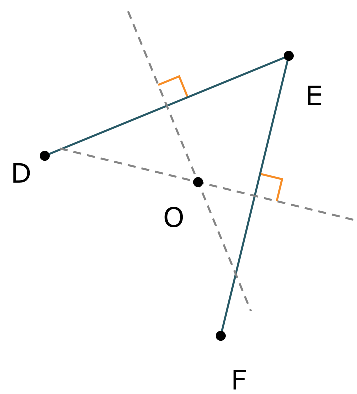
Your teacher will give you further directions about how to submit your work. You may be asked to upload the document, e-mail it to your teacher, or print it and hand in a hard copy.

Now, let's get started!

Answer Key

Step 1: Construct a circle through three points not on a line.

a. Points D, E, and F are not in a line. To construct a circle through points D, E, and F, begin by drawing line segments $\overline{DE}$ and $\overline{EF}$. Then construct the perpendicular bisectors of $\overline{DE}$ and $\overline{EF}$, and name the point of intersection of the perpendicular bisectors O. How do you know that point O is the center of the circle that passes through the three points? (10 points)



Possible Answers:

1. If you construct $\overline{DF}$, then you form △DEF. The definition of a circle is the set of all points in a plane that are equidistant from a point called the center. Therefore, for point O to be the center of the circle that passes through the three vertices of △DEF, point O needs to be equidistant from points D, E, and F. The concurrency of perpendicular bisectors of a triangle theorem states that the perpendicular bisectors of a triangle intersect at a point, called the circumcenter, that is equidistant from the vertices of the triangle. So by constructing the perpendicular of two sides of △DEF and finding the intersection point, we know by the theorem that this is the center of the circle that passes through points D, E, and F.

2. The definition of a circle is the set of points in a plane that are equidistant from a given point called the center. Therefore, for point O to be the center of the circle that passes through points D, E, and F, point O must be equidistant from each of the three points. By the perpendicular bisector theorem, all points on a perpendicular bisector of a segment are equidistant from the endpoints of that segment. That means that all points on the perpendicular bisector of $\overline{DE}$ are equidistant from points D and E. Likewise, all points on the perpendicular bisector of $\overline{EF}$ are equidistant from points E and F. The intersection of the perpendicular bisectors is the point that is equidistant from all three points. Therefore, O is the center of the circle that passes through points D, E, and F.

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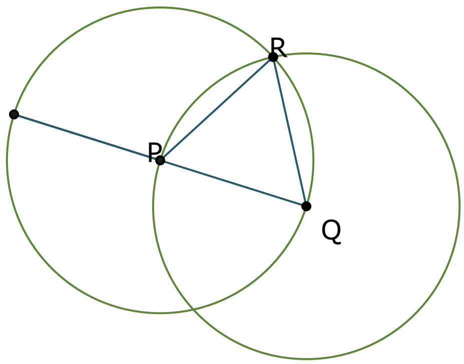
Step 2: Construct regular polygons inscribed in a circle.

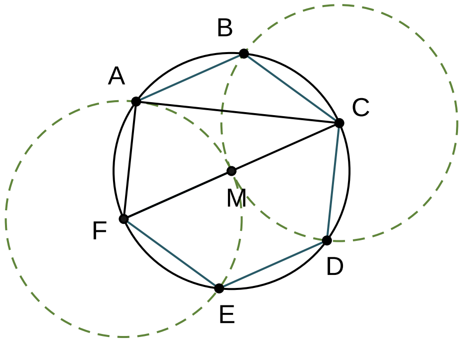
a. While constructing an equilateral triangle or a regular hexagon inscribed in a circle, you may have noticed that several smaller equilateral triangles are formed, like △PQR shown in the figure below. Explain why △PQR is an equilateral triangle. (5 points)



Possible Answer:

We know that PR = PQ because they are both radii of circle P. Likewise, we know that PQ = RQ because they are both radii of circle Q. By the transitive property, PR = PQ = RQ. Using the definition of congruence, $\overline{PR} \cong \overline{PQ} \cong \overline{RQ}$. Therefore, by the definition of equilateral triangle we can say that △PQR is an equilateral triangle.

b. The completed construction of a regular hexagon is shown below. Explain why △ACF is a 30˚-60˚-90˚ triangle. (10 points)



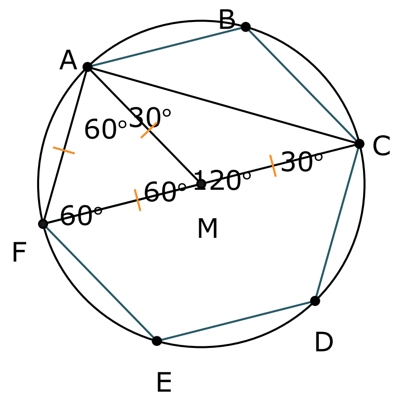
Possible Answers:

1. By construction, $\overline{FC}$ is a diameter of circle M, which means FAC is a semicircle. Because ∠FAC is an inscribed angle in semicircle FAC, angle FAC is a right angle because an angle inscribed in a semicircle is a right angle. That means that *m*∠FAC = 90° and that △ACF is a right triangle. Now, AF = FM because they are both radii of circle F. FM = MC because they are both radii of circle M. Therefore,

Answer Key

AF = FM = MC by the transitive property. By the segment addition postulate, FC = FM + MC. Using substitution, FC = AF + AF, or FC = 2AF. So the leg and the hypotenuse of the right triangle are in a ratio of 1:2, which only exists in a 30º-60º-90º triangle.

2. If you draw in segment AM, you can create △FAM and △AMC. Using the reasoning from part a, we know that △FAM is an equilateral triangle. That means that △FAM is equiangular, which means $m\angle AFM = m\angle FMA = m\angle MAF = 60°$. Because △FAM is equilateral, we know FA = AM = FM. Because both are radii of circle M, FM = MC. Therefore, we know that AM = MC by substitutuion, making △AMC isosceles. Notice that angles AMF and AMC form a linear pair. By the linear pair postulate, we know these angles are supplementary, which makes $m\angle AMC = 120°$. Because the base angles of an isosceles triangles are congruent, angles MAC and MCA must each be 30°. We now have shown that for △FAC, angle F is 60° and angle C is 30°, which makes angle A 90°, making △FAC a 30º-60º-90º triangle.



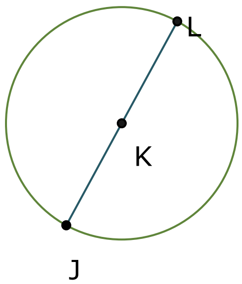
c. If you are given a circle with center C, how do you locate the vertices of a square inscribed in circle C? (5 points)

Possible Answer:

First, construct a diameter of circle C. The two endpoints will be two vertices of the square. Then, construct a perpendicular bisector of this diameter. This step ensures that the diagonals of the square are both congruent and perpendicular. Locate the intersections of the perpendicular bisector and the circle. These are the two other vertices of the square. Connect consecutive vertices to create the square.

Step 3: Construct tangent lines to a circle.

a. $\overline{JL}$ is a diameter of circle K. If tangents to circle K are constructed through points L and J, what relationship would exist between the two tangents? Explain. (5 points)



Possible Answer:

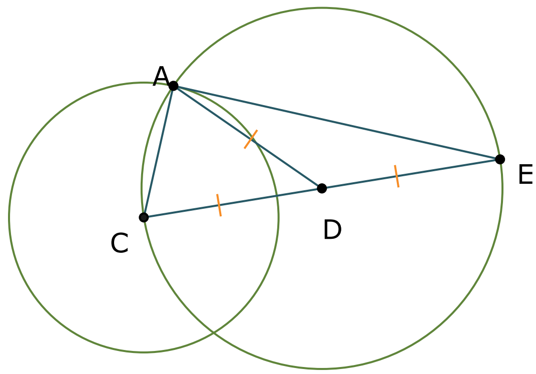
The two tangent lines would be parallel to each other. The lines tangent to points J and L would be perpendicular to $\overline{JL}$ because a tangent is perpendicular to a radius of a circle at the endpoint of the radius. Because the tangents are perpendicular to the same line, $\overline{JL}$, by the perpendicular transversal theorem, they are parallel to each other.

b. The construction of a tangent to a circle given a point outside the circle can be justified using the second corollary to the inscribed angle theorem. An alternative proof of this construction is shown below. Complete the proof. (5 points)

Given: Circle C is constructed so that CD = DE = AD; $\overline{CA}$ is a radius of circle C.

Prove: $\overline{AE}$ is tangent to circle C.

Answer Key

 **Statements**	**Reasons**
1. Circle C is constructed so that CD = DE = AD; $\overline{CA}$ is a radius of circle C.	1. Given
2. $\overline{CD} \cong \overline{DE} \cong \overline{AD}$	2. Definition of congruence
3. △ACD is an isosceles triangle; △ADE is an isosceles triangle.	3. Definition of isosceles triangle
4. $m\angle CAD + m\angle DCA + m\angle ADC = 180°$; $m\angle DAE + m\angle AED + m\angle EDA = 180°$	4. Triangle angle sum theorem
5. $\angle CAD \cong \angle DCA$; $\angle DAE \cong \angle AED$	5. Isosceles triangle theorem
6. $m\angle CAD = m\angle DCA$; $m\angle DAE = m\angle AED$	6. Definition of congruence
7. $m\angle CAD + m\angle CAD + m\angle ADC = 180°$; $m\angle DAE + m\angle DAE + m\angle EDA = 180°$	7. Substitution property
8. $2(m\angle CAD) + m\angle ADC = 180°$; $2(m\angle DAE) + m\angle EDA = 180°$	8. Addition
9. $m\angle ADC = 180° - 2(m\angle CAD)$; $m\angle EDA = 180° - 2(m\angle DAE)$	9. Subtraction property
10. ∠ADC and ∠EDA are a linear pair.	10. Definition of a linear pair

Answer Key

11. ∠ADC and ∠EDA are supplementary.	11. Linear pair postulate
12. $m\angle ADC + m\angle EDA = 180°$	12. Definition of supplementary angles
13. $180° - 2(m\angle CAD) + 180° - 2(m\angle DAE) = 180°$	13. Substitution property
14. $360° - 2(m\angle CAD) - 2(m\angle DAE) = 180°$	14. Addition
15. $-2(m\angle CAD) - 2(m\angle DAE) = -180°$	15. Subtraction property
16. $m\angle CAD + m\angle DAE = 90°$	16. Division property
17. $m\angle CAD + m\angle DAE = m\angle CAE$	17. Angle addition postulate
18. $m\angle CAE = 90°$	18. Substitution property
19. ∠CAE is a right angle.	19. Definition of right angle
20. $\overline{AE} \perp \overline{CA}$	20. Definition of perpendicular
21. $\overline{AE}$ is tangent to circle C.	21. Converse of the radius-tangent theorem