Probability Applications
Answer Key

Applying Probability Concepts

Did you know that all 50 states in the United States have some sort of state fair? Fairs and carnivals include a variety of games of chance that aren't typically "fair." What does it mean for a game to be fair? In this task, you'll analyze different scenarios to determine fairness. You will use probability and expected value to make these determinations. You will also use these concepts to make decisions.

As you complete the assignment, keep this question in mind:

How can probability be applied to decision making?

Directions

Complete each of the following tasks, reading the directions carefully as you go. Show all work where indicated, including inserting images of graphs. Be sure that all graphs or screenshots include appropriate information, such as titles, labeled diagrams, etc. If your word processing program has an equation editor, you can insert your equations here. Otherwise, print this activity sheet and write your answers by hand.

In addition to the answers you determine, you will be graded based on the work you show, or your solution process. So, be sure to show all your work and answer each question as you complete the task. Type all your work into this document so you can submit it to your teacher for a grade. You will be given partial credit based on the work you show and the completeness and accuracy of your explanations.

Your teacher will give you further directions about how to submit your work. You may be asked to upload the document, e-mail it to your teacher, or print it and hand in a hard copy.

Now, let’s get started!

Answer Key

1. Miguel is playing a game in which a box contains four chips with numbers written on them. Two of the chips have the number 1, one chip has the number 3, and the other chip has the number 5. Miguel must choose two chips, and if both chips have the same number, he wins $2. If the two chips he chooses have different numbers, he loses $1 (−$1).
   a. Let X = the amount of money Miguel will receive or owe. Fill out the missing values in the table. (Hint: The total possible outcomes are six because there are four chips and you are choosing two of them.) (5 points)
   Answer:

X_i	2	−1
$P(x_i)$	$\frac{1}{6}$	$\frac{5}{6}$

   b. What is Miguel's expected value from playing the game? (5 points)
   Answer:

   E(X) = value · $P(x_1)$ + value · $P(x_2)$ + value · $P(x_3)$ + ...

   $E(X) = 2\left(\frac{1}{6}\right) + (-1)\left(\frac{5}{6}\right)$

   $E(X) = -\frac{1}{2}$

   c. Based on the expected value in the previous step, how much money should Miguel expect to win or lose each time he plays? (5 points)
   Answer:
   Miguel should expect to lose $0.50 each time he plays.

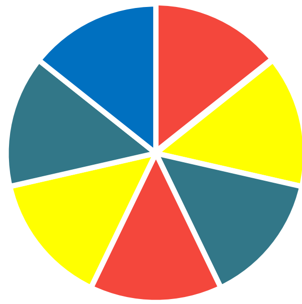
   d. What value should be assigned to choosing two chips with the number 1 to make the game fair? Explain your answer using a complete sentence and/or an equation. (5 points)
   Answer:

   In order for the game to be fair—i.e., the expected value = 0—the value for drawing two 1's should be $5.

   $E(X) = 5\left(\frac{1}{6}\right) + (-1)\left(\frac{5}{6}\right) = 0$

Answer Key

2. A game at the fair involves a wheel with seven sectors. Two of the sectors are red, two of the sectors are purple, two of the sectors are yellow, and one sector is blue.



Landing on the blue sector will give 3 points, landing on a yellow sector will give 1 point, landing on a purple sector will give 0 points, and landing on a red sector will give −1 point.

a. Let X = the points you have after one spin. Fill out the missing values in the table. (5 points)
Answer:

X_i	3	1	0	−1
$P(x_i)$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{2}{7}$

b. If you take one spin, what is your expected value? (5 points)
Answer:

$E(X) = \text{value}(x_1)(p_1) + \text{value}(x_2)(p_2) + \text{value}(x_3)(p_3) + \text{value}(x_4)(p_4)$

$E(X) = 3\left(\frac{1}{7}\right) + (1)\left(\frac{2}{7}\right) + (0)\left(\frac{2}{7}\right) + (-1)\left(\frac{2}{7}\right)$

$E(X) = \frac{3}{7}$

c. What changes could you make to values assigned to outcomes to make the game fair? Prove that the game would be fair using expected values. (10 points)
Possible Answers: (answers will vary)

i. Make blue sectors worth 2 points and red sectors worth −2 points.
$E(X) = 2\left(\frac{1}{7}\right) + (1)\left(\frac{2}{7}\right) + (0)\left(\frac{2}{7}\right) + (-2)\left(\frac{2}{7}\right)$

$E(X) = 0$

ii. Make blue sectors worth 2 points and purple sectors worth −1 points.

$E(X) = 2\left(\frac{1}{7}\right) + (1)\left(\frac{2}{7}\right) + (-1)\left(\frac{2}{7}\right) + (-1)\left(\frac{2}{7}\right)$

$E(X) = 0$

iii. Make the blue sector worth 0 points.

$E(X) = (0)\left(\frac{1}{7}\right) + (1)\left(\frac{2}{7}\right) + (0)\left(\frac{2}{7}\right) + (-1)\left(\frac{2}{7}\right)$

$E(X) = 0$

3. The point guard of a basketball team has to make a decision about whether or not to shoot a three-point attempt or pass the ball to another player who will shoot a two-point shot. The point guard makes three-point shots 30 percent of the time, while his teammate makes the two-point shot 48 percent of the time.

X_i	3	0
$P(x_i)$	0.30	0.70

X_i	2	0
$P(x_i)$	0.48	0.52

a. What is the expected value for each choice? (5 points)

Answer:

E(Shoot) = 3(0.30) = 0.90

E(Pass) = 2(0.48) = 0.96

b. Should he pass the ball or take the shot himself? Explain. (5 points)

Answer:
He should pass the ball to his teammate. There is a greater expected value for passing the ball.

Answer Key

4. Claire is considering investing in a new business. In the first year, there is a probability of 0.2 that the new business will lose $10,000, a probability of 0.4 that the new business will break even ($0 loss or gain), a probability of 0.3 that the new business will make $5,000 in profits, and a probability of 0.1 that the new business will make $8,000 in profits.
    a. Claire should invest in the company if she makes a profit. Should she invest? Explain using expected values. (5 points)
    Answer:

    Claire's expected value will be:

    $E(X) = (-10{,}000)(0.2) + (0)(0.4) + (5{,}000)(0.3) + (8{,}000)(0.1)$

    $E(X) = 300$

    So Claire should invest and expect to make $300 the first year from the new business.

    b. If Claire's initial investment is $1,200 and the expected value for the new business stays constant, how many years will it take for her to earn back her initial investment? (5 points)
    Answer:

    It will take Claire four years to break even from her initial investment.

Answer Key

5. Tanya is considering playing a game at the fair. There are three different ones to choose from, and it costs \$2 to play a game. The probabilities associated with the games are given in the table.

	Lose \$2	**Win \$1**	**Win \$4**
Game 1	0.55	0.20	0.25
Game 2	0.15	0.35	0.50
Game 3	0.20	0.60	0.20

   a. What is the expected value for playing each game? (5 points)
   Answer:
   E(Game 1) = −2(0.55) + 1(0.20) + 4(0.25) = 0.10

   E(Game 2) = −2(0.15) + 1(0.35) + 4(0.50) = 2.05

   E(Game 3) = −2(0.20) + 1(0.60) + 4(0.20) = 1.00

   b. If Tanya decides she will play the game, which game should she choose? Explain. (5 points)
   Answer:

   Tanya should play Game 2 because it is the only game where she is expected to make more money than she paid to play.