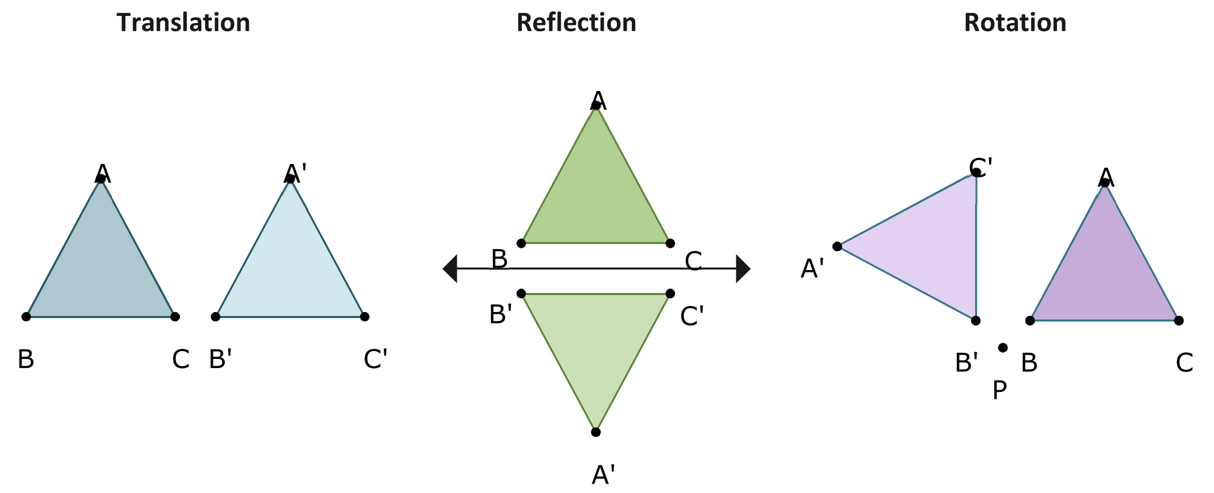
Geometric Proofs

Logical, deductive reasoning requires a systematic way of thinking that outlines steps of mathematical statements to take given information and prove a mathematical statement. This form of reasoning is the backbone for geometric proofs, utilizing definitions, postulates, theorems, and other properties of geometric figures to show that a statement is true. Many geometric proofs involve congruency statements, to be proven using isometric transformations or congruency theorems.



As you complete the task, keep these questions in mind:

How can rigid transformations be used to prove congruency? How can congruency theorems be used to prove congruency?

In this task, you will apply what you have learned in this unit to answer these questions.

Directions

Complete each of the following tasks, reading the directions carefully as you go. Be sure to show all work where indicated, including inserting images of graphs. Be sure that all graphs or screenshots include appropriate information such as titles, labeled diagrams, etc. If your word processing program has an equation editor, you can insert your equations here. Otherwise, print this activity sheet and write your answers by hand.

In addition to the answers you determine, you will be graded based on the work you show, or your solution process. So, be sure to show all your work and answer each question as you complete the tasks. Type all your work into this document, or print the document and show work by hand, so you can submit it to your teacher for a grade. You will be given partial credit based on the work you show and the completeness and accuracy of your explanations.

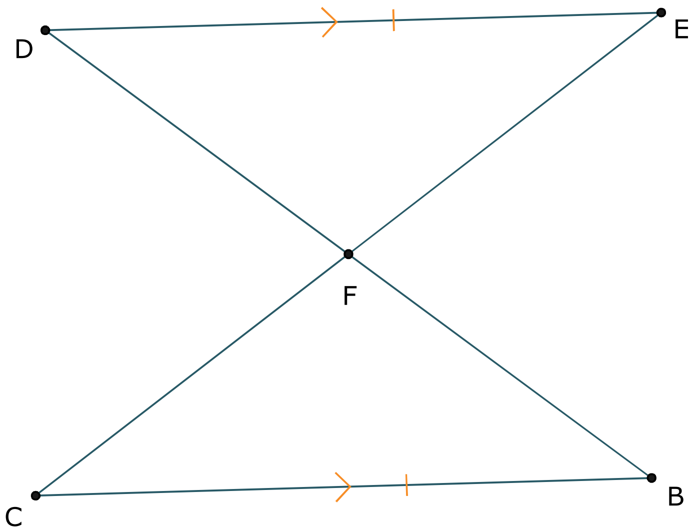
Your teacher will give you further directions about how to submit your work. You may be asked to upload the document, e-mail it to your teacher, or print it and hand in a hard copy.

Now, let's get started!

Part 1: Use transformations to prove congruency.

1. Given: $\overline{CB} \parallel \overline{ED}$; $\overline{CB} \cong \overline{ED}$

   Prove: $\triangle CBF \cong \triangle EDF$ using isometric (rigid) transformations.



Outline the necessary transformations to prove $\triangle CBF \cong \triangle EDF$ using a paragraph proof. Be sure to name specific sides or angles used in the transformation and any congruency statements.

(10 points)

Possible Answer:

We are given that $\overline{CB}$ is parallel and congruent to $\overline{ED}$. When two parallel lines are cut by a transversal, we know that the alternate interior angles are congruent. Thus, angle EDF is congruent to angle CBF, and angle DEF is congruent to angle BCF. Isometric transformations preserve angle measures, so angle EDF would have to map to angle CBF and angle DEF would have to map to angle BCF. We also know that segments CB and ED are congruent, so they will map to each other as well. If we set F as the center of rotation, then we can see that each vertex of the pre-image triangle EDF will form a straight line when connected to the center of rotation and then from the center of rotation to the corresponding vertex of the image triangle CBF. The image is a copy of the pre-image because rotations preserve lengths and angles. The center of rotation will stay the same, while vertex E maps to vertex C and vertex D maps to vertex B. Since there is a rigid transformation preserving angle measures and side lengths, triangle CBF and triangle EDF are congruent.

Students may draw in diagrams of each transformation.

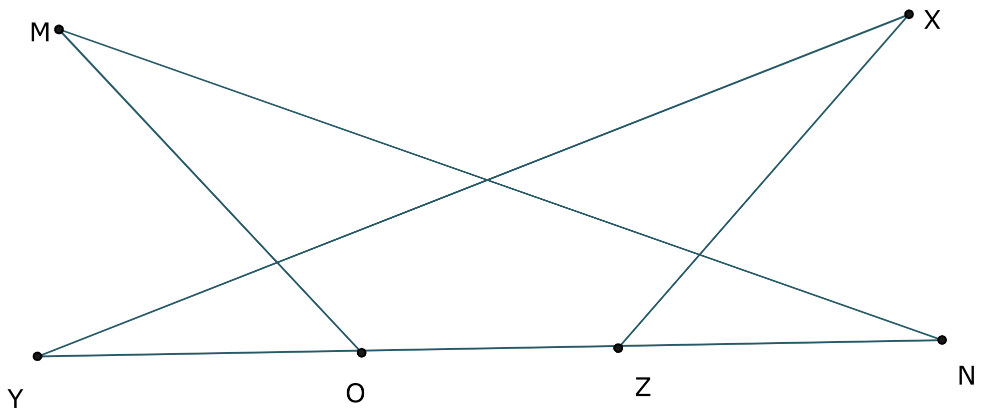
Teacher Guide

Part 2: Use congruency theorems to prove congruency.

1. The congruency of △MNO and △XYZ can be proven using a reflection across the line bisecting $\overline{OZ}$. However, this congruency can also be proven using geometric postulates, theorems, and definitions. Prove that the triangles are congruent using a two-column proof and triangle congruency theorems. (10 points)

Given: $\angle M \cong \angle X$
$\angle N \cong \angle Y$
$\overline{YO} \cong \overline{NZ}$

Prove: $\triangle MNO \cong \triangle XYZ$



Possible Answer:

Statements	Reasons
1. $\angle M \cong \angle X$	1. given
2. $\angle N \cong \angle Y$	2. given
3. $\overline{YO} \cong \overline{NZ}$	3. given
4. $YO = NZ$	4. definition of congruence

Teacher Guide

5. OZ = ZO	5. reflexive property
6. YO + OZ = NZ + ZO	6. addition property
7. YZ= YO + OZ NO = NZ + ZO	7. segment addition postulate
8. YZ = NO	8. Substitution
9. $\overline{YZ} \cong \overline{NO}$	9. definition of congruence
10. $\triangle MNO \cong \triangle XYZ$	10. AAS Congruency theorem (1, 2, and 9)

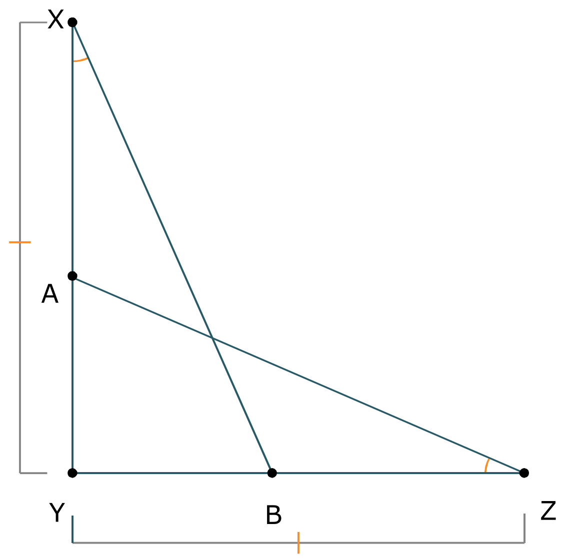
Teacher Guide

2. Use the diagram and given information to answer the questions and prove the statement.

Given: $\angle X \cong \angle Z$

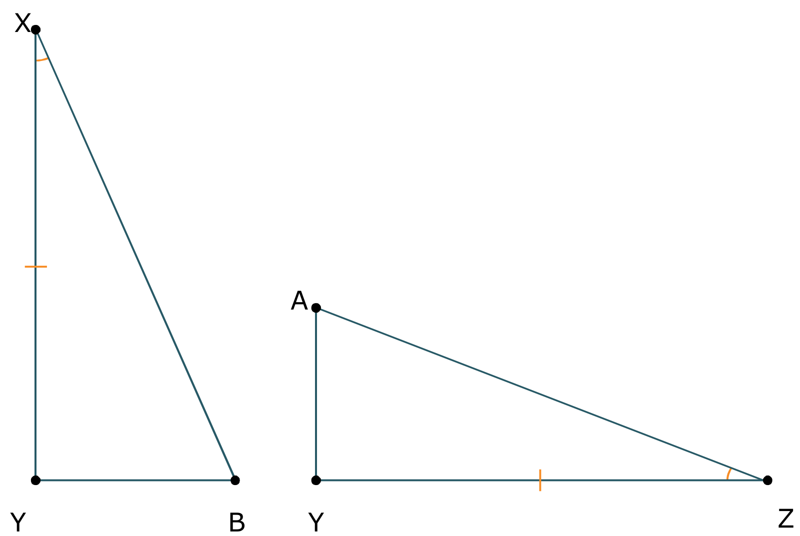
$\overline{XY} \cong \overline{ZY}$

Prove: $\overline{AZ} \cong \overline{BX}$



a. Re-draw the diagram of the overlapping triangles so that the two triangles are separated. (5 points)

Possible answer:



b. What additional information would be necessary to prove that the two triangles, $\triangle XBY$ and $\triangle ZAY$, are congruent? What congruency theorem would be applied? (5 points)

Possible answers:

- We would need to know that angle Y is congruent to itself to use the ASA congruency theorem.
- We would need to know that angle A is congruent to angle B to use the AAS congruency theorem.
- We would need to know that side BX is congruent to side AZ to use the SAS congruency theorem.
- We would need to know that side XB is congruent to side AZ and that side BY is congruent to side YAY to use the SSS congruency theorem.

c. Prove $\overline{AZ} \cong \overline{BX}$ using a flow chart proof. (10 points)

Possible answer:

$\angle X \cong \angle Z$
$\overline{XY} \cong \overline{ZY}$

Given

$\angle Y \cong \angle Y$

Reflexive Property

$\triangle XYB \cong \triangle ZYA$

ASA $\cong$ Theorem

$\overline{AZ} \cong \overline{BX}$

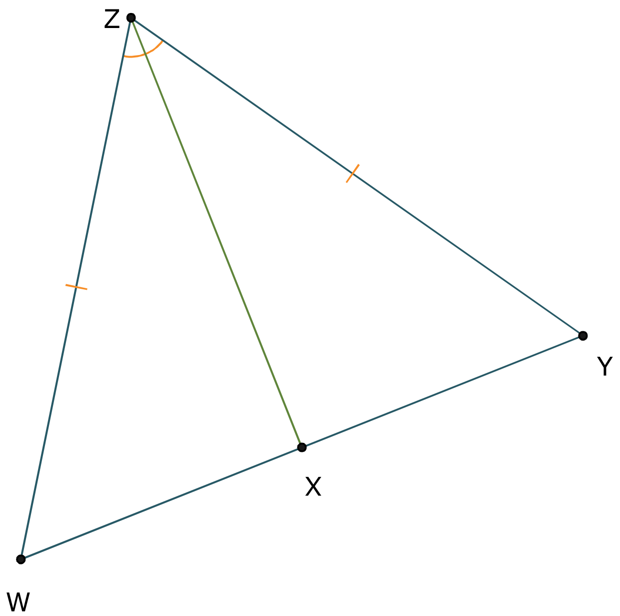
CPCTC

Teacher Guide

Part 3: Choose a proof method.

1. Use a paragraph, flow chart, or two-column proof to prove that $\overline{ZX}$ is the perpendicular bisector of side $\overline{WY}$.

Given: $\angle WZX \cong \angle YZX$; $\overline{ZW} \cong \overline{ZY}$
Prove: $\overline{ZX}$ is a perpendicular bisector of $\overline{WY}$.



Possible answer:
Paragraph proof:

We know that angle WZX is congruent to angle YZX and that side ZW is congruent to side ZY. Using the reflexive property, we can state that $\overline{ZX}$ is congruent to $\overline{ZX}$. Thus, by the SAS congruency theorem, triangle WZX is congruent to triangle YZX. We know that corresponding parts of congruent triangles are congruent, so angle ZXY is congruent to angle ZXW. That is, they have equal angle measures. These two angles are also a linear pair because they make up the straight line segment WY, so we know that $m\angle ZXY + m\angle ZXW = 180°$. We can substitute $m\angle ZXY$ for $m\angle ZXW$ because their measures are equal, giving us $2(m\angle ZXW) = 180°$. Solving, we find that $m\angle ZXY = m\angle ZXW = 90°$.This implies that $\overline{ZX}$ is perpendicular to $\overline{WY}$ by the definition of perpendicular. Finally, using CPCTC again we know that $\overline{WX} \cong \overline{YX}$. Therefore, X is the midpoint of $\overline{WY}$ and $\overline{ZX}$ is the perpendicular bisector of $\overline{WY}$.

Teacher Guide

Flow Chart proof:

$\angle WZX \cong \angle YZX$

$\overline{ZW} \cong \overline{ZY}$

Given

$\overline{ZX} \cong \overline{ZX}$

Reflexive Property

$\triangle WZX \cong \triangle YZX$

SAS Congruency Theorem

$\angle ZXY \cong \angle ZXW$

CPCTC

$m\angle ZXY \cong m\angle ZXW$

Definition of Congruent

$\angle ZXY$and $\angle ZXW$ are a linear pair

Definition of Linear Pair

$m\angle ZXY + m\angle ZXW = 180$

Property of Linear Pair

$m\angle ZXY + m\angle ZXY = 180$, or $2(m\angle ZXY) = 180$

Substitution

$m\angle ZXY = 90$

Division Property

$\overline{ZX}$ is perpendicular to $\overline{WY}$

Definition of Perpendicular

$\overline{WX} \cong \overline{YX}$

CPCTC

X is the midpoint of $\overline{WY}$

Definition of Midpoint

$\overline{ZX}$ is a bisector of $\overline{WY}$

Definition of Bisector

$\overline{ZX}$ is the perpendicular bisector of $\overline{WY}$

Definition of Perpendicular Bisector

Teacher Guide

Two-column proof:

Statements	Reasons
1. ∠WZX ≅ ∠YZX	1. given
2. $\overline{ZW} \cong \overline{ZY}$	2. given
3. $\overline{ZX} \cong \overline{ZX}$	3. reflexive property
4. △WZX ≅ △YZX	4. SAS congruency theorem
5. ∠ZXY ≅ ∠ZXW	5. CPCTC
6. *m*∠ZXY = *m*∠ZXW	6. definition of congruent
7. ∠ZXY and ∠ZXW are a linear pair	7. definition of linear pair
8. *m*∠ZXY + *m*∠ZXW = 180	8. property of linear pair
9. *m*∠ZXY + *m*∠ZXY = 180	9. substitution
10. 2(*m*∠ZXY) = 180	10. addition
11. *m*∠ZXY = 90	11. division property
12. ∠ZXY is a right angle	12. definition of right angle
13. $\overline{ZX} \perp \overline{WY}$	13. definition of perpendicular
14. $\overline{WX} \cong \overline{YX}$	14. CPCTC
15. $\overline{ZX}$ is a perpendicular bisector of $\overline{WY}$	15. definition of perpendicular bisector

Teacher Guide

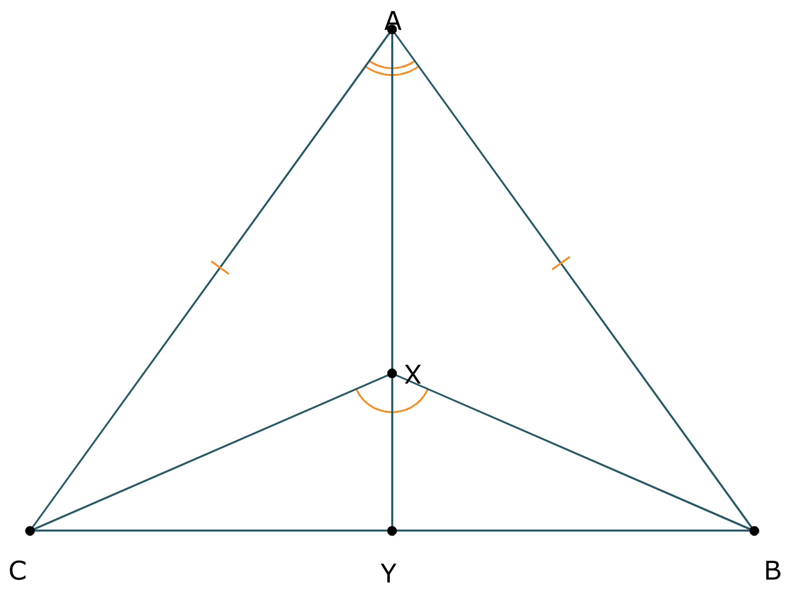
2. Use a paragraph, flow chart, or two-column proof to prove the angle congruency.

   Given: ∠CXY ≅ ∠BXY

   ∠CAX ≅ ∠BAX

   $\overline{AC} \cong \overline{AB}$

   Prove: ∠XCY ≅ ∠XBY



Possible answer:

Paragraph proof:

We are given that angle CXY is congruent to angle BXY, angle CAX is congruent to angle BAX, and side AC is congruent to side AB. We see that side AX is congruent to itself by the reflexive property. Thus, triangle AXC is congruent to triangle AXB by the SAS congruency theorem. Using CPCTC we know that side CX is congruent to side BX. Also, side XY is congruent to itself by the reflexive property. Therefore, triangle YXB is congruent to triangle YXC using the SAS congruency theorem. We can conclude that angle XCY is congruent to angle XBY by CPCTC.

Teacher Guide

Flow chart proof:

$\angle CAX \cong \angle BAX$
$\angle CXY \cong \angle BXY$

Given

$\overline{AX} \cong \overline{AX}$

Reflexive

$\triangle AXC \cong \triangle AXB$ → $\overline{CX} \cong \overline{BX}$

SAS | CPCTC

$\overline{XY} \cong \overline{XY}$

Reflexive Property

$\angle XCY \cong \angle XBY$ ← $\triangle YXB \cong \triangle YXC$

CPCTC | SAS

Teacher Guide

Two-column proof:

Statements	Reasons
1. ∠CXY ≅ ∠BXY	1. given
2. ∠CAX ≅ ∠BAX	2. given
3. $\overline{AC} \cong \overline{AB}$	3. given
4. $\overline{AX} \cong \overline{AX}$	4. reflexive property
5. △AXC ≅ △AXB	5. SAS congruency theorem
6. $\overline{CX} \cong \overline{BX}$	6. CPCTC
7. $\overline{XY} \cong \overline{XY}$	7. reflexive property
8. △YXB ≅ △YXC	8. SAS congruency theorem
9. ∠XCY ≅ ∠XBY	9. CPCTC